The Use of Manipulatives in the Teaching of Maths

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This article will concentrate on the language, methodology and materials used in the teaching of the basic concepts and algorithms to children who are experiencing difficulty in the learning of mathematics.

My first contact with children with learning difficulties in mathematics is in first class and the main objective would be that they would be able to count, recognise and write numbers – one to fifty and do simple addition and subtractions. When they first attend learning support, most children are able to count, recognise and write numbers less than ten. However, occasionally there are some who don't. One such child was 'James', a very loveable boy who had severe difficulty in literacy and numeracy, but was very eloquent in the spoken language. When we first met, he was able to recognise the numbers one, two and three, but not the others. Whenever I showed him the number four, he would take a wild guess. I spent a number of weeks trying various methods to get him to recognise the number four. One day, in total exasperation, I said to him, (with an understanding smile on my face) 'James, you will break my heart'. The next day, I showed him the number four again for the nth time. He looked at me with a broad smile on his face and said 'is that the one that breaks your heart Sir?" I replied, "Yes" and to my amazement and eternal relief 'James' said 'four'.

The incident set me off on the road to teaching recognition of numbers through association e.g. four 'knock on the door', 'six a bundle of sticks' etc. So beside the number four, I would draw a door and beside the number six a bundle of sticks and so on with other numbers. This works reasonably well but it can still be a hard struggle as those children who are experiencing such a difficulty in First Class are usually very weak.

As a forerunner to addition, children use separate number discs rather than a fixed number line. This means that they have to construct their own number line each day before commencing work. This gives them a physical sense of the ordinality of number i.e. that one comes before two, four comes after three etc. As well as that, they spend a long time walking the number line using large floor mats with the numbers written on them. They start by taking one jump at a time e.g. 5+1, 5+2, 6+3 etc.

In the beginning they are allowed to use the complete number line but in order to reduce their dependency on the number line, they are required to use number lines with some numbers concealed. As they progress more and more numbers are concealed e.g.

Step 1: 5+3 = ☐ 

Step 2: 2+3 = ☐ 

Step 3: 4+3 = ☐ 

The same procedure is followed with subtraction. If one was to wait for proficiency in adding smaller numbers (6+3) before moving onto adding and subtracting larger numbers (26+13) then the wait would be a very long one indeed. Hence place value and the understanding of the composition of larger numbers is introduced before that goal has been achieved.

Place value is introduced with the use of lollipop sticks and the children being asked to make tens. Firstly they are given ten single lollipop sticks and asked to make a bundle of ten. They discover that ten single sticks make one ten and there are no ones left over.

We write ten as:

T U

1 0

With eleven they make one bundle of ten and there is one unit left over.
Eleven is written as:

T U

1 1

They proceed along those lines until they discover that twenty units make two tens and no unit and is written as:

T U

2 0

Square pieces of cardboard are used as placeholders. The same procedure is followed with larger numbers.

The next step is to pick up a given number e.g. 13 or 24 in the ‘quickest way possible’ using lollipop sticks. At this stage it becomes apparent fairly quickly which children have or have not a good concept of number composition. For example when concept is established, a child when asked to pick up 13 will use one bundle of ten and three single units, whereas the child where the concept is still weak will proceed to pick up thirteen by using units only.

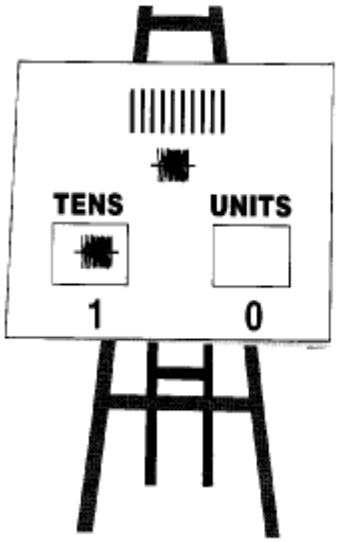
The above exercises can also be done using money. Children like variety and in order to hold their interest teachers must use a variety of concrete materials. Money is very attractive to children. Most teachers will have come across children with severe difficulty in mathematics but who will never be short-changes when it comes to money. Some such children run a very successful paper round ands some go on the

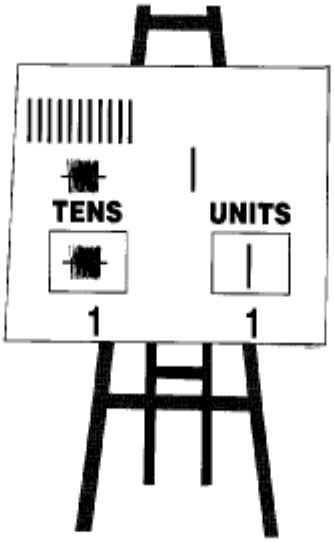
be successful business people! Money also lends itself much better than lollipop sticks to making larger numbers in the higher classes e.g. 12,234 – the euro coins can be used for the units, the ten euro and hundred euro notes for tens and hundred and cheques for the thousand and ten thousands. All three pieces of equipment are magnetic and can be easily manipulated on a magnetic board. The materials can be manipulated either by the teacher or selected children in full view of the whole class. This in turn leads to the development of the mathematical language that is such a vital part of mathematics itself.

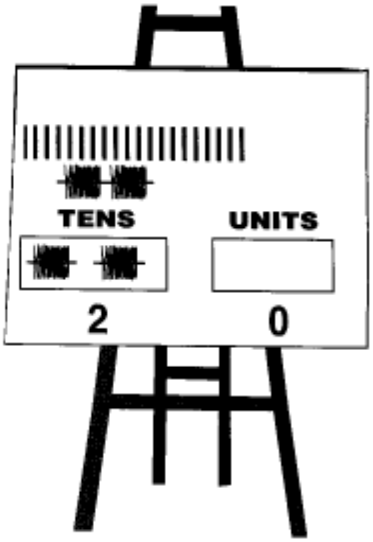
The idea of the magnetic board arose while giving workshops to various groups of teachers throughout the country. It was often rightly pointed out to me that , laudable as the use of concrete materials may be, their usage was a lot less cumbersome for me as a remedial teacher working with small groups of children than for the ordinary classroom teacher with 30+ children. Hopefully the magnetic board will make the use of concrete materials more amenable for the classroom teacher and will lead to far more use being made of them in the ordinary class. This in turn should make concept formation easier and more enjoyable for the child, especially the less able child.

The following is an example of the language used and the activity involved using the magnetic board and the equipment that accompanies it, when dealing with place value.

BEGINNING PLACE VALUE



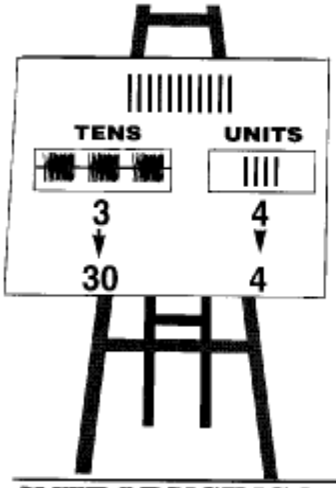




Count out Ten.
Make a ten. How many are left?
Ten makes one ten and no unit.
We write one in the tens place
and zero in the units place.

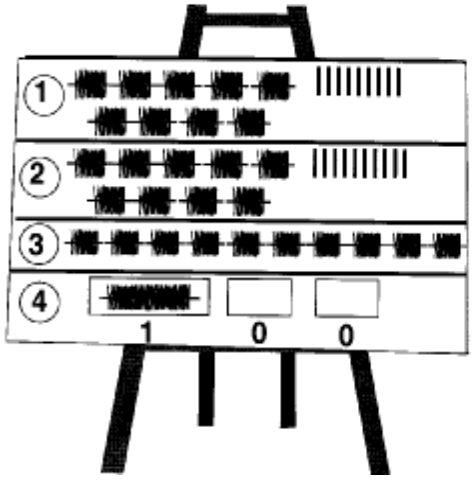
Repeat with other numbers as shown

MAKING NUMBERS



Using the tens and the units make thirty four.
What is the three worth?
What is the four worth?
Which would you prefer, the three or the four?
Etc. with other numbers.

INTRODUCING NEW NUMBERS

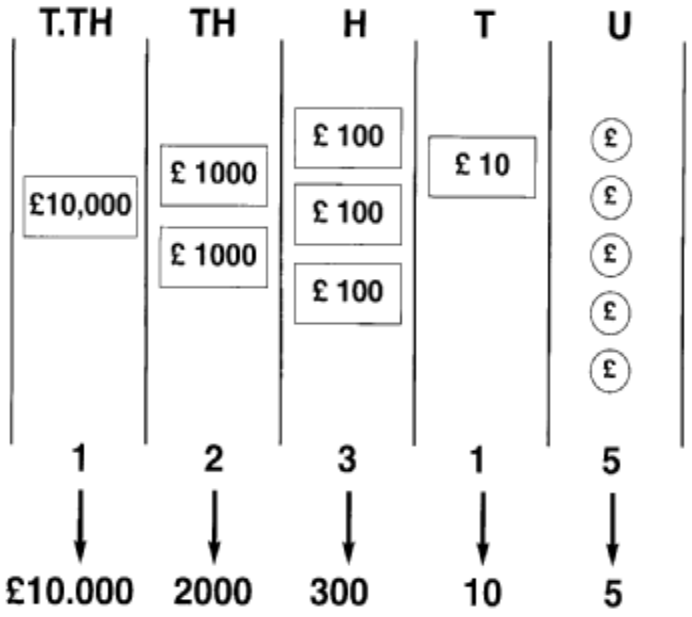


(1) Make 99
(2) Add one more.
We now have ten ones.
Make a ten.
(3) We now have ten tens.
(4) Make a hundred.
Put it in the hundreds place.
Write one hundred.
Make one hundred and three.
Write one hundred and three
Same with 113 130 etc.

LARGE NUMBERS

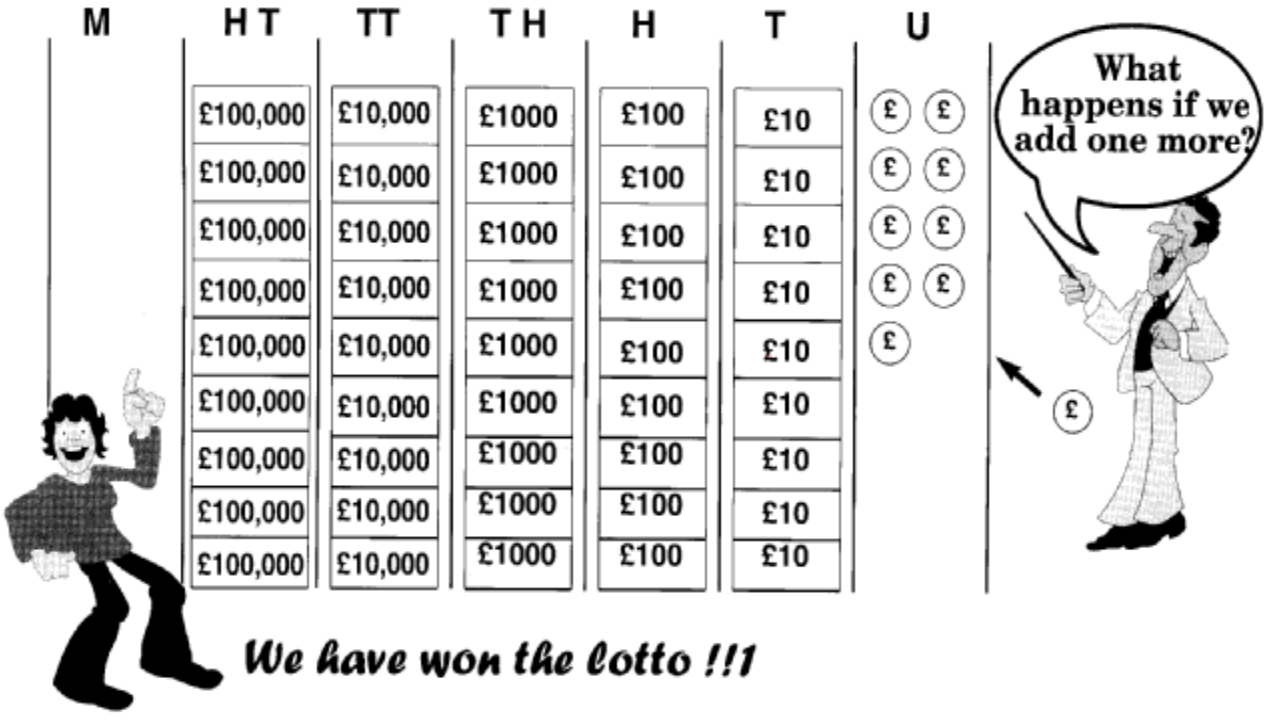
Children like to talk about millions and trillions and 'gillions" but in fact have a very poor understanding of numbers beyond a thousand. They also experience considerable difficulty in writing larger numbers. A typical mistake would be 12000500 for twelve thousand five hundred.

Money is ideal for building concept from hundreds, to thousands, to tens of thousands, to hundreds of thousands, to millions.



Make, then write twelve thousand three hundred and fifteen

What is the value of each digit?



Returning to the lower classes, when children have a good grasp of number composition (2 digit) and the basics of the base ten system, they are introduced to the addition of two-digit number. The scope of this article will not permit detailing the highly structured method, which must be followed in order to make the learning situation more amenable to the child with learning difficulties. Rather it will focus on one or two particular steps on the road to mastering a particular concept.

In the case of addition the focus will be on addition with renaming, which is a problematic area.

First of all, let's have a look at the most common mistakes that occur.

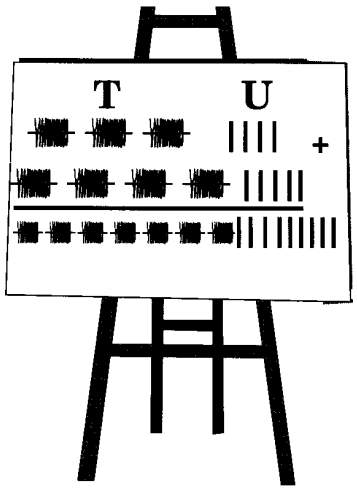
2 5	←	no idea of	→	2 5
2 8		place value		2 8
1 7				5 1 2

3 4	←	forgot to	→	2 4
2 8		carry		4 5
5 2				7 9

It is obvious from these mistakes that most computation errors occur because of lack of concept. In order to make concept formation easier for the child with learning difficulties, it is necessary to use concrete materials. For example, in doing the sum 23 + 38, they are asked to make the number using either lollipop sticks or money. When they put the units together to make eleven, they then make a ten with the sticks and place it with the other tens. At this stage, the teacher does the recording on the boards while discussing each move with the children. The following is an outline of the activity involved and the language used.

ADDITION

$$\begin{array}{r} 34 \\ +45 \\ \hline \end{array}$$



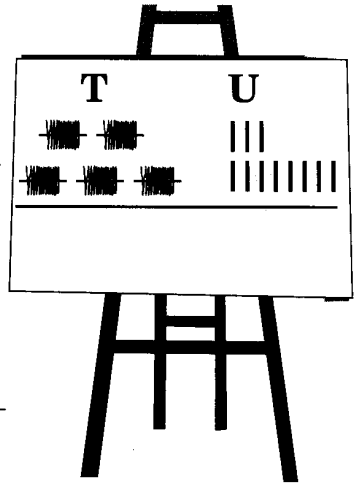
Make thirty four
Make forty five
Add the units
Add the tens
How many altogether?

Being asked to make the numbers physically gives the child a very good understanding of number composition, ordinality of number and place value.

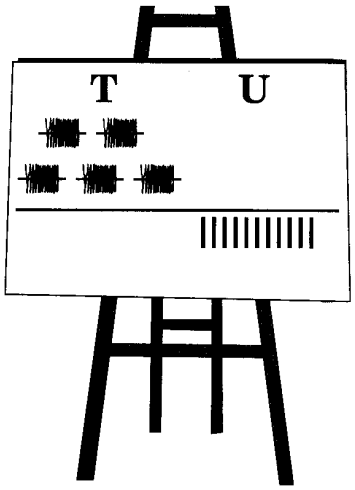
ADDITION WITH RENAMING

$$\begin{array}{r} 23 \\ +38 \\ \hline \end{array}$$

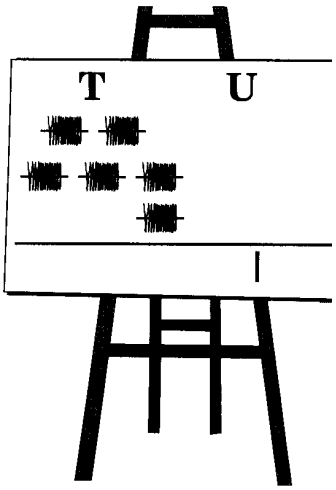
$$\begin{array}{r} 23 \\ +3_{1}8 \\ \hline 1 \end{array}$$



Make twenty three.
Make thirty eight.

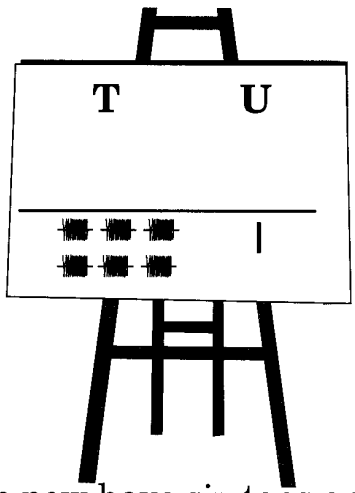


Add the units.



Make a ten and place in the tens place.
Now add the tens.

$$\begin{array}{r} 23 \\ +3_{1}8 \\ \hline 61 \end{array}$$



We now have six tens and one unit. We have sixty one.

While most children will cope with addition without renaming many will experience difficulty with addition with renaming unless it is done concretely.

Many examples are done with the children using concrete materials only (no written work) while the teacher is at the board deriving the appropriate language form the children and doing the recording. When the teacher feels that the concept has been sufficiently established, the children do the written algorithms using concrete materials. Then at the appropriate time the concrete material is withdrawn but is returned to again when mistakes occur.

The above steps are followed whenever a new algorithm or concept is introduced.

(a) Children use concrete material only (no writing). Teacher at the board doing the recording and deriving the appropriate language.
(b) Children do written work using concrete material.
(c) Children do written work without concrete materials.
(d) Return to concrete materials when mistakes occur.

These steps will not be referred to again but the reader may take it that they are the steps that are followed no matter what concept or algorithm is being discussed. There is a strong emphasis on the use of concrete materials. They are used in three of the four main steps. This should not be surprising since according to Piaget, the majority of children in the Primary School are at the ‘concrete operational stage’ and need concrete materials in order to form concepts. It should also be noted that in the revised curriculum developed by the NCCA ‘activity based learning is stressed and there is less emphasis on pencil and paper calculations. The use of manipulatives is an important element in all programmes from Infants to Sixth’. (Comhairle Information Bulletin of NCCA January 1996).

Let us now turn out attention to subtraction and the difficulties that chidlren face in gaining mastery of this concept.
Subtraction is a problematic area for various reasons.

Teachers use different language e.g.

9
- 4

Some say nine take away four as this is the language used by children when doing 9 – 4. Others say four from nine. There is also a difference in the methods used. Some teachers do composition, others use the 'borrow pay back' method while other s do equal addition. Further confusion arises when parents do not understand the method used by the teacher and try to teach the child their own way at home.

Children can also use the wrong language when doing an algorithm e.g. 54 – 25 = ☐ The child says four from five leaves one. He/she may use the right language but the wrong answer – five from four leaves one. Or they may say five take four leaves one. The confusion caused by different methods and language could be alleviated by agreeing a common language and methodology throughout the school.

In this article, we will focus only on the most problematic area of subtraction, that is, subtraction with renaming. Having observed children at work over the years, the most common mistakes and the ones that keep recurring are:

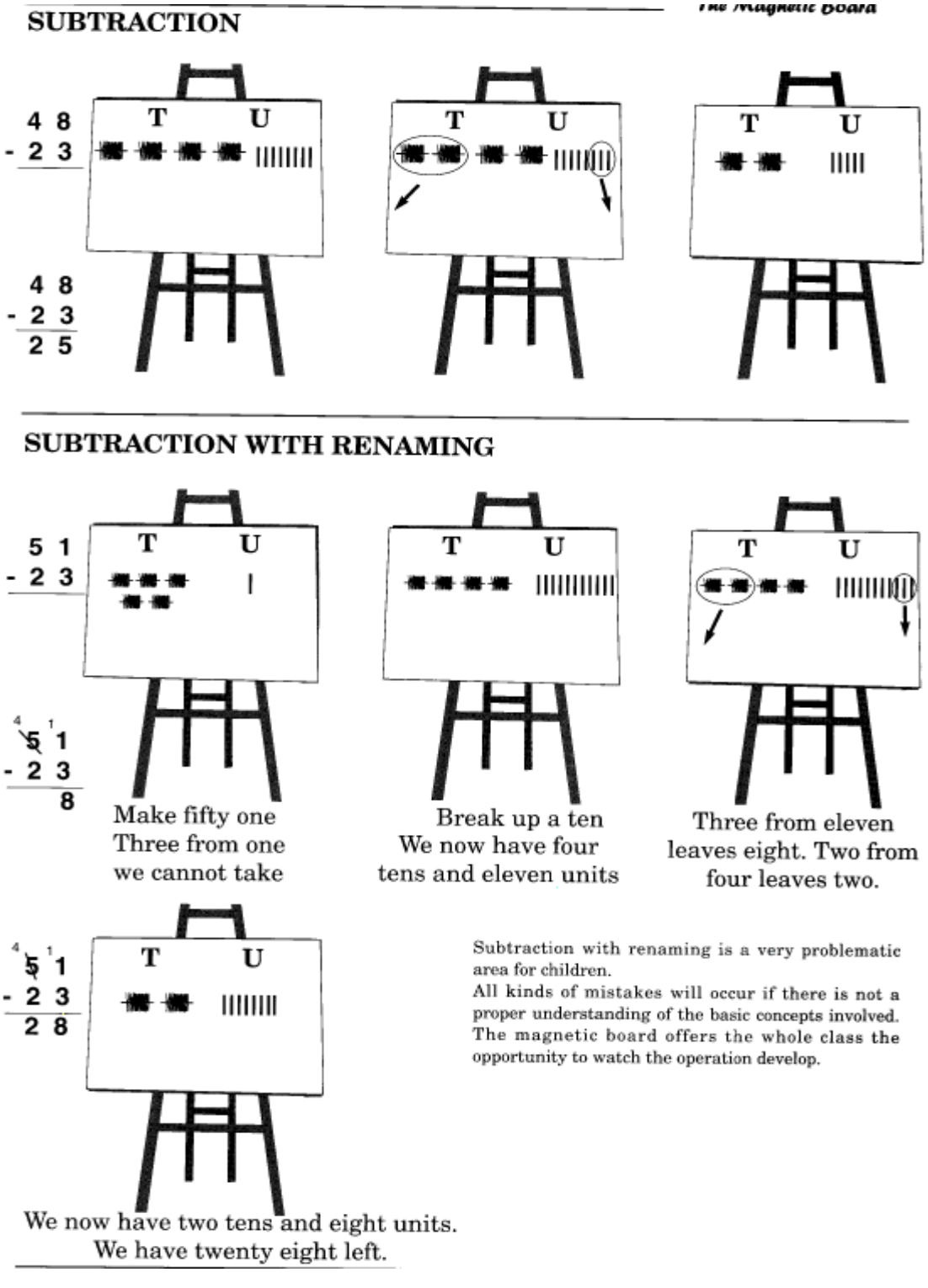
6 4 child says	7 3 forgets to
- 2 8 four from 8	- 2 5 "cross out"
4 4	5 8

7	1 1 1
8 7 "crosses out"	2 0 0 does not understand
- 2 5 when no need	- 1 4 5 what is happening

3 9 9 9
4 0 0 0 does not understand
- 1 2 4 5 the concept

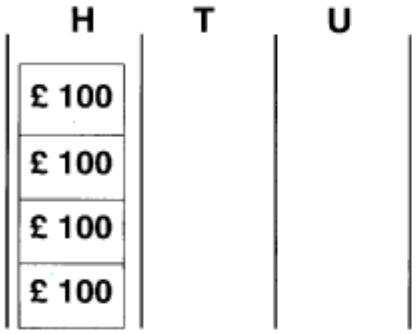
"Crossing out" means nothing to the child and in my own context chidlren are not allowed to use the words. If they are using lollipop sticks, they say, "we break up a ten". If using money, they say, " we go tot eh bank and get a ten euro coins for the ten euro note". The following is an example of the use of manipulatives and the language used when dealing with this most difficult area:

Subtraction



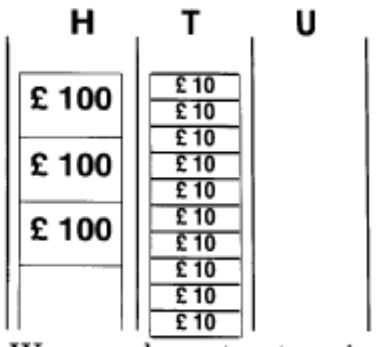
MORE DIFFICULT SUBTRACTION

400
-245



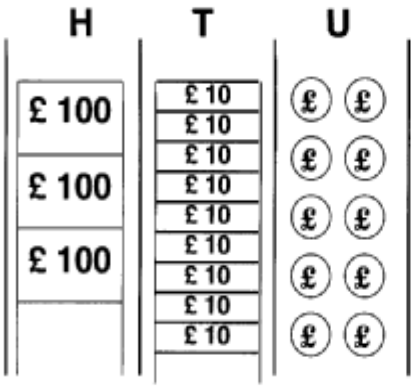
$^{3}\not{4}^{1}\not{0}0$
-245

Zero take five we cannot do
We break up a ten but there is no ten there.
We break up a hundred.



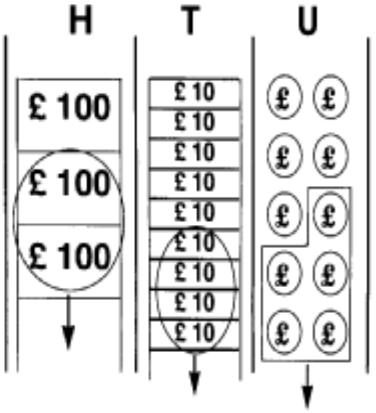
We now have ten tens in the tens place but we still have no units.
We break up a ten.

$^{3}\not{4}^{9}\not{0}^{1}0$
-245



$^{3}\not{4}^{9}\not{0}^{1}0$
$-\not{2}\not{4}\not{5}$
155

We now have nine tens in the tens place and ten units in the units place.



Ten take five leaves five.
Nine take four leaves five.
Three take two leaves one.
We have 155 left.

If this operation is not done concretely typical mistakes will be $^{1\,1}400$ -245 or $^{9\,9}400$ -245

The child is only guessing and is unsure because she/he does not understand the operation.

With large numbers it's even more necessary to use concrete materials e.g. 4000 -1345

Money is convenient to use and meaningful to the child

There will be a considerable amount of time spent on step one (teacher recording, chidlren manipulating material) referred to above before going on to steps two and three. If mistakes still occur after step three, they are corrected using the manipulatives.

This article has endeavoured to portray how manipulative and concrete materials are an integral part of a teacher's paraphernalia when trying to get a concept across or to understand the reasoning behind the various movements in an algorithm. The scope of this article only allowed us to examine how addition, subtraction and place value might be approached. However, it must be stated that concrete materials and manipulatives are even more necessary in other areas e.g. fractions and decimals.

Fractions, for example are a new number system and therefore concepts in fractions are very difficult to grasp. For the child with learning difficulties it's impossible without the use of concrete materials.

As has been stated already, the revised curriculum places great emphasis on the use of concrete materials and manipulatives. Perhaps not before its time.

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